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REVIEW

An Introduction to Finite Element Analysis. By D. H. NORRIE and G. DE VRIES. Academic Press, 1978. 301 pp. $16.00 or £11.35.

Finite Element Analysis in Fluid Dynamics. By T. J. CHUNG. McGraw-Hill, 1978. 378 pp. $38.00.

Finite Elements in Fluids. Edited by R. H. GALLAGHER, O. C. ZIENKIEWICZ, J. T. ODEN, M. M. CECCHI and C. TAYLOR. Wiley, 1978. Vol. III, 396 pp. £17.50.

These three books represent a spectrum of hard-back offerings for fluid mechanicians interested in the finite element (FE) method. The first is not, of course, focused on fluids but is an introduction to the method. The second book is a complete text devoted to finite elements and fluids in general. Finally, the third is a continuation of a series and follows from the 1976 *Second International Conference on Finite Elements in Flow Problems*.

In their book Norrie and de Vries have focused on working 'from the particular to the general' with a view toward developing 'a basic understanding of the finite element method rather than providing a comprehensive coverage of its many applications'. The first chapter outlines the basic concepts of the finite element method, while Chapter 2 uses a specific example of a variational formulation for heat flow in two-dimensions to show how things go together when one solves a particular problem. The description and examples of the formulation of the matrices is particularly good. However, the variational formulations are introduced without any justification. A brief physical interpretation would have helped considerably. Chapters 3 to 6 cover computer programming, boundary conditions, various elements, operations on the system of element equations, and aspects of computation. These early chapters are excellent for beginners and give even finite element aficionada good concrete examples. The programs and worked-out examples are very helpful to the learning process. This section of the text makes it a worthwhile complement to the traditional FE texts in one's library (for example, Norrie and de Vries' earlier book or those by Huebner and Zienkiewicz).

The remainder of Norrie and de Vries' book is basically descriptive and often qualitative, a completely different thrust from the early chapters.

Chapter 7 on variational calculus includes a good discussion of the required order of trial functions. Chapter 8 describes convergence, completeness, and conformity, but does not demonstrate how accurate actual computations are, i.e. there are no comparisons of computed and analytical solutions. Chapter 9 on elements and their properties is a routine description, but gives only a cursory outline of the *use* of the important isoparametric elements. Chapter 10 on equation solvers and programming techniques is a good description of what is available and the merits of each method. Again, direct comparisons of accuracy or speed from actual example computations are missing. The selected applications of Chapter 11 are valuable because the formulation and matrix set-ups are clearly laid out. Unfortunately, the currently dominant weighted-residual methods are discussed relatively briefly and not until Chapter 12.

In sum, the greatest value of the Norrie and de Vries book lies in the detailed

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descriptions in Chapters 2 to 6. The remaining portion of the text is primarily descriptive, albeit useful.

The book by T. J. Chung presents the finite element method as a means of solving boundary and initial value problems in fluid dynamics. Chung's primary objective is to teach the reader how to solve practical problems in fluid dynamics and his secondary objective is to provide a mathematical treatment sufficient to establish the validity of the calculated solutions. As discussed below, the book has a broad and general coverage of both the mathematics and the practicalities of the finite element method as applied to fluid dynamics. However, in many areas the book is mathematically abstract and sophisticated and it lacks the motivational paragraphs of physical substance that are normally included in a book intended for the 'uninitiated student' among others. Accordingly, readers should have previous grounding both in the finite element method through study of an elementary text and in tensor mathematics of functional analysis.

Chapter 1 begins with an introduction to mathematical notations and functional analysis, variational methods, the method of weighted residuals, and the finite element in one dimension. The presentation of the mathematical preliminaries (tensors, functional analysis, etc.) is of a review rather than in instructional nature, but the finite element presentation goes from the beginning through a completely worked out example. [As an aside, the speed with which the finite element method is developing is seen by citing a statement on page 41: 'Galerkin and least squares methods are well adapted to finite element applications, whereas the methods of moments and collocation do not lend themselves to direct applications to the finite element method' and then noting that in July 1978 (*Second International Conference on Finite Elements in Water Resources*), Pinder, Frind and Cellia presented a method for the solution of two-dimensional potential flow problems which combined orthogonal collocation and finite elements.]

Chapter 2 describes finite interpolation function functions, giving a catelogue and description of various elements, but little on how or why to choose among them. Indeed, the Hermite polynomials are introduced as providing continuity of derivitives at common nodes; however, it is not clearly explained that most of the other elements in the chapter do not provide continuity of derivitives at common nodes or why this is important.

Chapter 3 discusses local and global forms of the finite element equations, boundary conditions, the solution of the equations for steady state and time dependent problems, and the mathematical properties of finite elements, including error estimates. This excellent chapter shows nicely the relationship between the finite element method and various solution techniques that one must use, for example, in nonlinear problems or time-dependent problems.

Chapter 4 describes fluid dynamics preliminaries in an abstract and complex manner without a real physical context. Chapter 5, then, becomes the first experience with actual fluid flows and considers incompressible flow. The development here is excellent and the author has done a superb job in setting up the examples from basics so that the reader can follow them carefully. The description of what goes on at the boundaries and in the matrices is also well done. It is valuable to have computer programs (in appendix B) made available.

Chapter 6 focuses on compressible flow. Here Chung introduces a weighted-residual

method in the global sense, using the concept of an invariant inner product as he did in Chapter 5. The conclusions reached are consistent with more traditional, physical reasoning and the technique itself is a valuable and consistent way of approaching the problem. The remainder of Chapter 6 follows the previous pattern with excellent formulation of the problems, including the boundary conditions, etc., and it is a good review with some useful techniques regarding shocks and transonic flow being described.

Chapter 7 covers miscellaneous flow problems and is basically descriptive. Such things as finite element solution of the governing equations for diffusion, magnetohydrodynamics, and rarefied gas dynamics are described.

The final book in this group, the third volume in a series on finite elements in fluids, is intended to be a collection of major contributions arising from the *Second International Conference on Finite Elements in Flow Problems* held in 1976. The editors asked the authors to update and revise their articles which were originally presented in 1976 with the hope that the chapters of the book would represent the status of the fast-moving finite element field at the time the text went to print. The effort was generally successful.

There are 21 papers in this volume. Six contain essentially new formulations or concepts (including a penalty-function formulation; development of restricted variational forms and application of a least-squares weighted-residual approach to the Navier–Stokes equations; development of dual mixed-hybrid finite element methods; the use of finite elements and variational methods to find eigenvalues in fluid stability analyses, and the application of optimal control theory to the solution of finite element problems). Two papers are examinations of specific features of finite element calculation (namely, optimal weighting for upwind schemes with finite elements and examination of the effects of mass lumping and the accuracy of the finite element method in advection-dominated flows). Six are review articles in the areas of nonconforming (non-conventional) FE methods; transonic aerodynamics; variational methods for free surface, separated and wake flows; the application of finite elements in the context of the method of variational inequalities; and finally, finite elements in general (Chapter 21). This chapter is essentially an annotated bibliography to about 200 recent references in the areas of potential flow, creeping flow, incompressible viscous flow, compressible flow, porous media flow, dispersion, circulation problems, and miscellaneous other areas. Finally, seven papers describe specific applications of existing methods to particular problems ranging from the Navier–Stokes equations to tidal flow, transonic flow, groundwater flow and plasmas in Tokamaks. Surprisingly, none of the 21 papers focuses on the nature of the finite elements themselves.

While a few papers used variational approaches, most papers employed a weighted-residual formulation. Otherwise, the scope of ideas for problem formulation is broad and readers will find this volume useful in a number of areas. As an example, Chapters 9 to 12 give a good overview of transonic flow problem analysis by the finite element method. Chapters 9 and 12 are reviews covering, essentially, formulation for various cases and computational methods, respectively. Chapter 10 describes calculations for a range of formulations which ultimately employ standard finite element procedures. Chapter 11 gives the formulation of three-dimensional multibody problems as equivalent well-posed optimal-control problems.

Among the seven specific application papers, there is a tendency for the work reported on to be already in the literature. The papers serve, then, to bring together the results of several previous publications. Chapter 13, Environmental Flow Analysis for Stratified Conditions, is a case in point. Both the theory and basic results were presented in three papers during 1975 and 1976 in the *Americal Society of Civil Engineers Proceedings*. Indeed, parts of the text and several of the figures were taken verbatim from these publications.

The three volumes reviewed here form an interesting and worthwhile complementary set.

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